SCANNER DATA-BASED PANEL PRICE INDEXES

CHEN ZHEN, ERIC A. FINKELSTEIN, SHAWN A. KARNS, EPHRAIM S. LEIBTAG, AND CHENHUA ZHANG

We construct panel price indexes using retail scanner data that allow comparisons of consumption cost across space and time. Two types of panel indexes are examined: the rolling-window panel extensions of the multilateral Cave-Christensen-Diewert index with the Törnqvist index as its elements, and of the multilateral Gini-Eltetö-Köves-Szulc index using the Fisher ideal index as its elements. The rolling window method maintains the nonrevisability of published index numbers while allowing index numbers for new periods and locations to be calculated and the basket of items to be updated. Meanwhile, the multilateral structure of price comparison eliminates significant downward drift in standard chained indexes. Using county-level bilateral and panel indexes based on retail beverage scanner data, we experimentally adjust for purchasing parity the portion of Supplemental Nutrition Assistance Program (SNAP) benefits that participants spend on beverages. Accounting for temporal and spatial cost differences causes over 2% of SNAP allotment spent on beverages to be reallocated, or approximately a 5% change in allotment on average for a county. About 90% of the relocated SNAP fund is to adjust for spatial differences in food cost. We also compare SNAP allotments implied by the retail scanner data indexes with those implied by indexes based on the USDA Quarterly Food-at-Home Price Database (QFAHPD). The treatment of unit values and product quality may have contributed to the significant differences observed between the retail scanner data indexes and the QFAHPD indexes.

Key words: Panel price index, scanner data, SNAP allotment adequacy.

JEL codes: C43, D12, Q18.

Tracking time series price variation is of great interest to governments, firms, researchers, and consumers because of the importance of monitoring and adjusting for inflation. Equally important is the comparison of prices across geographical areas. Several studies have examined the economic implications of spatial price differences. For example,

Albouy (2009) argued that mortgage and local-tax deductions partially index taxes to the local cost of living and, thereby, reduce federal tax distortion of employment distribution across cities. In another example, Jolliffe (2006) reported a complete reversal of the rankings of the prevalence, depth, and severity of poverty between metropolitan and nonmetropolitan areas after adjusting for geographical differences in living cost. These results have important implications for poverty-reduction policies because eligibility for many federal assistance programs is linked to the official poverty guidelines, which are currently the same across all areas in the continental United States. In addition, as investigated in several recent research studies and reports (Dubay, Wheaton, and Zedlewski 2013; Gregory and Coleman-2013; U.S. Jensen Government Accountability Office 2014), the benefit amount for program participants could potentially be adjusted for living-cost differences based on the recipient's location. Incorporating these potential program

Amer. J. Agr. Econ. 00(0): 1–19; doi: 10.1093/ajae/aay032

© The Author(s) 2018. Published by Oxford University Press on behalf of the Agricultural and Applied Economics Association. All rights reserved. For permissions, please email: journals.permissions@oup.com

Chen Zhen is a UGA Athletic Association associate professor at the University of Georgia. Eric A. Finkelstein is a professor in the Duke-NUS Graduate Medical School. Shawn A. Karns is a research analyst at RTI International. Ephraim S. Leibtag is an assistant administrator at the Economic Research Service, USDA. Chenhua Zhang is a scientific data analyst at ICF.

The views expressed in this article are those of the authors and do not reflect the official view of the USDA or ERS. The authors thank the editor, Tim Beatty, and three anonymous referees for valuable comments and suggestions, and Melanie Ball for her research assistance during the early phase of this project. Zhang assisted solely with programs for computer codes when he was an assistant professor of statistics at the University of Georgia and only accessed the USDA Quarterly Food-at-Home Price Database (QFAHPD) for this research. Grant support from the National Heart, Lung, and Blood Institute, National Institute of Health (1R21HL108121) is gratefully acknowledged. Request for county-level price index data by beverage category may be sent to: czhen@uga.edu.

changes requires price indexes that properly measure price differences across space.

A panel price index allows price comparisons between locations in different time periods. Most existing price index databases are either spatial or temporal but not both despite the potential value of tracking spatial and temporal price changes simultaneously. It is possible to combine time series price indexes with spatial price indexes at a particular point in time to create panel price indexes. However, doing so may distort relative prices between locations at different points in time. In addition, compared with the long time series data on national price indexes, official statistics on within-country spatial price comparisons are only starting to emerge. For example, the U.S. Bureau of Labor Statistics and Bureau of Economic Analysis (BEA) have been releasing the consumer price index (CPI) and personal consumption expenditure price index, respectively, for many decades, while the official regional price parities (RPPs) that measure cost-of-living differences across states and metropolitan areas have been only available from the BEA since 2008 (BEA 2018). By contrast, international organizations such as the Organisation for Economic Co-operation and Development (OECD) and World Bank have made between-country price comparisons to calculate purchasing power parity exchange rates since the 1970s (OECD 2018; World Bank 2018).

A primary reason that panel price indexes have not been more readily available is due to limited data. Relatively few datasets collect time series price and quantity data for a large number of products at multiple locations that are readily available and useful for research and policy purposes. However, this is beginning to change with increasing accessibility of large-scale retail scanner data to the research community. Retail scanner data collect barcode-level quantity and dollar sales data at the point of sales for most grocery items, which represent about 40% of total expenditures in the CPI (Broda and Weinstein 2010). The simultaneous observation of quantity and price allows researchers calculate superlative price indexes to (Diewert 1976), which are more desirable than fixed-weight indexes as measures of consumption cost because the former account for item-level substitutions caused by relative price changes. Moreover, in contrast to sampling prices on a limited number of products at different points of time as is done in the construction of most official price indexes, scanner data offer continuous surveillance of both prices and quantities for the universe of products sold at retail outlets (Feenstra and Shapiro 2003).

This study compared eight scanner databased price indexes, including four variants of two new panel price indexes. In doing so, we contribute to the literature in two ways. First, Ivancic, Diewert, and Fox (2011) and de Haan and van der Grient (2011) demonstrated that the multilateral GEKS (Gini 1931; Eltetö and Köves 1964; Szulc 1964) can be applied to time series scanner data to calculate highfrequency price indexes that are superior to standard chained price indexes. We extended their approach by operationalizing the Gini-Eltetö-Köves-Szulc (GEKS) index and the multilateral Caves, Christensen, and Diewert (CCD; 1982) price indexes for panel scanner data. To ease calculation of panel indexes, we provide a SAS routine that has been optimized for big data applications.

Second, the majority of existing research uses one of two sources for regional food pri-The Council for Community and ces. Economic Research (C2ER) publishes quarterly prices of about 57 consumer goods and services, 29 of which are food and beverage products in about 400 U.S. urban areas. The prices are sampled in the first week of each quarter from 5 to 10 retail establishments. As a result, C2ER prices are less accurate than price indexes based on scanner data that track both prices paid and purchase quantity. The USDA Economic Research Service Quarterly Food-at-Home Price Database represents an important improvement over the C2ER food prices by providing unit values for 52 food groups and 35 market areas based on Nielsen household scanner data (Todd et al. 2010). However, the classical concern about unit value bias caused by quality difference (Deaton 1988) may apply to at least some food groups in the database. The current study represents the beginning of an effort to construct a database of panel price indexes (instead of unit values) for all foods at home at a finely disaggregated level and relatively high geographical resolutions (e.g., county or metropolitan statistical area). The utility of such a panel database, once complete, goes beyond adjusting food costs for federal food assistance programs. Researchers could benefit from the levels of food category disaggregation and geographical resolution to better identify food demand parameters and potential causal relations between nutritionrelated diseases and food costs.

The remainder of this paper is structured as follows. In the next section, we introduce the panel extensions of the multilateral GEKS and CCD indexes. Then we illustrate with an application to create county-level nonalcoholic beverage price indexes, which are used to experimentally adjust food benefits from the Supplemental Nutrition Assistance Program (SNAP) for temporal and county differences in cost of living. We subsequently examine how results of the SNAP analysis would change if, instead of retail scanner data, the Quarterly Food-at-Home Price Database is used to construct the price indexes. The penultimate section summarizes the results and provides recommendations, while the final section notes the limitations and suggests avenues for future research.

Price Index Formulas

A large number of candidate price index formulas can be used with scanner data to compare consumption costs across space and time. We chose four fixed-base bilateral indexes commonly used in the temporal index number literature—Paasche, Laspeyres, Fisher ideal (Fisher 1922), and Törnqvist (1936)—and two panel extensions of the GEKS and CCD multilateral indexes from the spatial index literature.¹ The Paasche and Laspeyres indexes were chosen because of their popularity in applied research, while Fisher ideal, Törnqvist, GEKS, and CCD were evaluated because of their superlative properties in the sense of Diewert (1976).

To simplify notation for panel price comparisons, let an entity be a unique combination of location and time. For example, the same location (e.g., a county) in period t and period t + 1 is considered as two distinct entities in our index formulas. The Laspeyres price index comparing entity j with base 0 is

(1)
$$P_L^{0j} = \frac{\sum\limits_{\nu \in \nu_{0j}} p_\nu^J q_\nu^0}{\sum\limits_{\nu \in \nu_{0j}} p_\nu^0 q_\nu^0}$$

where p_{ν}^{j} is the price of product ν in entity j; p_{ν}^{0} and q_{ν}^{0} are the base price and quantity of product ν , respectively; and ν_{0j} denotes the common set of items sold in both base 0 and entity j. The Passche price index is defined as

(2)
$$P_P^{0j} = \frac{\sum\limits_{\nu \in \nu_{0j}} p_{\nu}^j q_{\nu}^j}{\sum\limits_{\nu \in \nu_{0j}} p_{\nu}^0 q_{\nu}^j}$$

where q_v^j is the quantity of product v in entity j. The Fisher ideal price index is the geometric mean of Laspeyres and Paasche indexes:

$$(3) \quad P_F^{0j} = \sqrt{P_L^{0j} \times P_P^{0j}}.$$

The Törnqvist index is defined as

(4)
$$P_T^{0j} = \exp\left\{0.5\sum_{\nu\in\nu_{0j}}(s_{\nu}^0 + s_{\nu}^j)\ln(p_{\nu}^j/p_{\nu}^0)\right\}$$

where s_{v}^{0} and s_{v}^{j} are budget shares of product v in base 0 and entity j, respectively. The Laspeyres and Paasche indexes provide an upper and lower bound for the true (but unobservable) cost-of-living index, respectively (Konüs 1924). One can derive exact cost-of-living indexes conditional on functional form assumptions. For example, assuming that the cost function is arbitrarily twice differentiable linear homogenous, an index formula is superlative if it is exact for a flexible functional form that is a second-order approximation to this cost function (Diewert 1976). The Fisher Ideal and Törnqvist indexes are superlative in this sense, while Laspeyres and Paasche are not. The Törngvist index is exact for the translog total or unit cost function, and the Fisher ideal index is exact for the quadratic mean of order two unit cost function (Diewert 1976).

Multilateral indexes designed originally for spatial price comparisons are transitive. That is, the index ratio between locations j and kremain unchanged whether price levels in the two locations are compared directly or indirectly through an intermediate location l. Although bilateral indexes are not transitive, the bilateral superlative indexes can be used

¹ Fixed-base indexes are also known as direct indexes, whose values are calculated by comparing different periods or locations directly with a fixed base period or location. This is in contrast to chained indexes, which have a moving base served as the chain link between a new period or location and the previous period or location. A fixed-base index by definition is based on a fixed basket of goods, while a chained index can be constructed based on a fixed basket or a flexible basket that updates as new data come in (Ivancic, Diewert, and Fox 2011).

as elements to construct transitive spatial indexes (Diewert 1996; Hill 1997). The multilateral GEKS index is designed as

(5)
$$P_{GEKS}^{0j} = \prod_{l=0}^{M_j} (P_F^{0l} \times P_F^{lj})^{1/(M_j+1)}.$$

Each elementary index inside equation (5) compares the price levels in entity *j* with the base via entity *l*. In spatial price comparisons, M_i is equal to the number of geographical locations excluding the base. The GEKS index is the geometric mean of all possible pairwise comparisons between entities. The multilateral CCD index P_{CCD}^{0j} is created by substituting P_T for P_F in the GEKS index. The base can be an arbitrarily selected entity or an artificially constructed average entity. It converts the (less useful) matrix of bilateral indexes between all possible pairs of entities l and *j* into a (more useful) vector of multilateral indexes, one for each entity relative to the base, the choice of which does not change the ratio of indexes between entities (Deaton and Dupriez 2011).

Rolling-Window Panel Indexes

Multilateral indexes such as GEKS and CCD can be used without modification to compare prices simultaneously across space and time. Several studies have done this with relatively aggregated commodity-level data. Hill (2004) constructed yearly GEKS, CCD, and other price indexes based on price and expenditure data for 82 commodity headings for a panel of 15 European Union countries over the 1995–2000 period. At least two complications arise, however, if scanner data are used for panel price comparison. First, scanner data for many consumer good categories contain thousands of unique barcodes. The large number of items can quickly render applications of the original GEKS or CCD indexes computationally prohibitive due to the large number of bilateral comparisons used as building blocks. Second, equation (5)requires recalculating index numbers for all entities as scanner data for additional locations or later periods become available. This is apparently unacceptable for most statistical agencies because they would never be able to finalize the index numbers.

In their seminal work, Ivancic, Diewert, and Fox (2011) proposed using multilateral indexes with a rolling window to construct time series chained price indexes based on scanner data. Chained temporal indexes are preferred to fixed-basket indexes because the former continuously update the product basket to reflect product entry and exit. However, an unintended consequence of chaining is that the index does not return to its base even when item-level prices go back to their base levels (Forsyth and Fowler 1981), a phenomenon known as chain drift. This can be particularly prominent in high frequency (such as weekly) data where temporary price cuts and the resulting stockpiling behavior by shoppers are more pronounced. Nevertheless, chain drift may still be present at monthly frequency (de Haan and van der Grient 2011). A standard multilateral index such as the GEKS or CCD index does not suffer from chain drift due to its perfect concordance with the transitivity property. Although transitivity does not hold exactly under Ivancic, Diewert, and Fox's (2011) rolling-window modification of the multilateral index, their method is largely free from any significant chain drift because comparisons within each rolling window are transitive (Ivancic, Diewert, and Fox 2011). By specifying a rolling window within which multilateral comparisons are conducted and the new entity is chain-linked to existing entities, one does not revise index numbers for existing entities in or before the rolling window using Ivancic et al.'s method.

Ivancic Diewert, and Fox's (2011) rollingwindow multilateral index was originally developed with temporal price comparison in mind. However, in principle it can be used to construct panel price indexes free of significant chain drifts. We construct the rollingwindow panel GEKS index as follows. Let I_t be the set of entities in the *t*th period. The artificial base entity I_0 consists of item-level prices and quantities equal to their per-entity averages over the first T periods (e.g., full year) of the sample. Let $I_{t,\tau}$ include the complete set of entities between periods t and τ $(t \leq \tau)$, that is, $I_{t,\tau} = I_t \cup I_{t+1} \cup \ldots \cup I_{\tau-1} \cup I_{\tau}$. Here, $I_{t,\tau}^i$ denotes a subset of $N_{t,\tau}^i$ entities (including entity *i* itself) in set $I_{t,\tau}$ that have at least one item in common with those of entity $i \ (i \in I_{t,\tau}).^2$

² It is possible, although a low probability even using retail scanner data, that some entities sell completely different products from entity *j*. In this case, it would not be possible to calculate elementary bilateral superlative indexes between *j* and these entities.

For an entity *i* in the first *T* periods, that is, $i \in I_{1,T}$, its panel rolling-window (RW) GEKS index is identical to the standard GEKS comparing prices between *i* and all entities between base 0 and period *T* (inclusive):³

(6)
$$P_{RWGEKS}^{0i} = P_{GEKS}^{0i}$$

= $\prod_{l \in I_{0,T}^{i}} (P_{F}^{0l} \times P_{F}^{li})^{1/N_{0,T}^{i}}$.

We now motivate the RWGEKS for entities after the first *T* periods. Let $P_{GEKS,T+1}^{0j}$ be the standard GEKS index for an entity *j* in period *T* constructed based on data between periods 1 and *T* + 1. Then

(7)
$$P_{GEKS,T+1}^{0j} = \prod_{l \in I_{0,T+1}^{j}} (P_{F}^{0l} \times P_{F}^{lj})^{1/N_{0,T+1}^{j}}.$$

Let k be an entity in period T + 1. The standard GEKS index for entity k using data from periods 1 through T + 1 is

(8)
$$P_{GEKS}^{0k} = \prod_{l \in I_{0,T+1}^k} (P_F^{0l} \times P_F^{lk})^{1/N_{0,T+1}^k}.$$

Dividing equation (8) by equation (7) yields

$$(9) \quad \frac{P_{GEKS}^{0k}}{P_{GEKS,T+1}^{0j}} = \frac{\prod_{l \in I_{0,T+1}^{k}} \left(P_{F}^{0l} \times P_{F}^{lk}\right)^{1/N_{0,T+1}^{k}}}{\prod_{l \in I_{0,T+1}^{j}} \left(P_{F}^{0l} \times P_{F}^{lj}\right)^{1/N_{0,T+1}^{j}}}.$$

It is tempting to use the right-hand side of equation (9) as the chain link to update the GEKS. However, there are still three issues to resolve. First, there is no guarantee that $I_{0,T+1}^k = I_{0,T+1}^j$ because of differences in the basket of products sold between entity *k* and *j*. The chain link as it stands in equation (9) involves bilateral comparisons within sets $I_{0,T+1}^k$ and $I_{0,T+1}^j$ and $I_{0,T+1}^j$ that require substantial computing resources if the two sets have a lot of

ten as $P_{RWGEKS}^{i} = P_{GEKS}^{i} = \prod_{l \in I_{1,T}^{i}} (P_{F}^{li})^{1/N_{1,T}^{i}}$.

entities. Second, except *j* being an entity one period prior to entity *k*, we have not said how *j* could be chosen from all possible entities in that period. Third, the denominator $P_{GEKS,T+1}^{0j}$ still uses data from T + 1, although link entity *j* is in *T*. All three issues are addressed by the panel RWGEKS index. The RWGEKS when *j* is in *T* and *k* in T + 1 is constructed as

(10)
$$P_{RWGEKS}^{0k} = P_{GEKS}^{0j}$$

 $\times \prod_{l \in \{I_{T+1-\kappa,T+1}^{j} \cap I_{T+1-\kappa,T+1}^{k}\}} (P_{F}^{lk}/P_{F}^{lj})^{1/N_{T+1-\kappa,T+1}^{j,k}}$

where κ $(1 \le \kappa \le T)$ is the number of preceding periods included in the rolling window of width $\kappa + 1$, and $N_{T+1-\kappa,T+1}^{j,k}$ is the number of elements in the set $\{I_{T+1-\kappa,T+1}^{j} \cap I_{T+1-\kappa,T+1}^{k}\}$. Equation (10) is more convenient than equation (9) because P_{GEKS}^{0j} is already computed by period T for routine price index reporting, but $P_{GEKS,T+1}^{0j}$ is constructed in T + 1 for the sole purpose of creating a chain link for period T + 1 price indexes. Finally, the RWGEKS index for entities in periods T + 2 and later is constructed as

(11)
$$P_{RWGEKS}^{0k} = P_{RWGEKS}^{0j} \times \prod_{l \in \{l_{\tau-\kappa,\tau}^{i} \cap I_{\tau-\kappa,\tau}^{k}\}} (P_{F}^{lk}/P_{F}^{lj})^{1/N_{\tau-\kappa,\tau}^{j,k}}$$

where $k \in I_{\tau}$, $j \in I_{\tau-1}$, and $\tau \ge T + 2$.⁴ To construct the rolling window CCD (RWCCD) index, we would substitute the elementary Törnqvist index P_T for the Fisher ideal index P_F in the RWGEKS index.

Intransitivity and Choice of Link Entity⁵

In this section, we discuss the transitive property of RWGEKS and RWCCD. Proofs are

³ It is neither necessary to choose the mean over entities in the first *T* periods as the artificial base, nor required to define a base for a panel index. However, referencing the panel price index to a base = 1 is in keeping with the convention on production of most official time-series price statistics and, therefore, may be easier to conceptualize for users unfamiliar with panel indexes. For example, the standard reference base period for current U.S. CPI series is 1982–84 = 100. Without a base, equation (6) is write the standard price of the standard price of the standard set of the standard price of

⁴ Due to temporal product turnover and spatial product unavailability, the product set used to construct P_F^k may not perfectly overlap with that of P_F^k . Lamboray and Krsinich (2015) noted that this may introduce bias, and proposed an intersection approach that used only the common set of products among entities *j*, *l*, *k* in the creation of P_F^k and P_F^j . These authors found this modification to work well in their application to consumer electronics data. We applied their method to the panel indexes (denoted intRWGEKS and intRWCCD) and found (*a*) each panel index and its intersection variant to be highly correlated (correlation coefficient > 0.98); and (b) the intersection method narrows the difference in index number between indexes of different rolling-window width.

⁵ We are indebted to an anonymous reviewer for bringing issues discussed in this section to our attention.

provided in supplementary online appendix A. As noted in Ivancic, Diewert, and Fox (2011), a consequence of the rolling-window approach to updating multilateral indexes is that the resulting indexes are no longer exactly transitive over the full sample. This means the relative price between two entities when compared directly is different from their ratio when compared through a third entity. So in theory, RWGEKS and RWCCD indexes could potentially be subject to chain drift. However, chain drift is unlikely to be significant in practice given a sufficiently wide window length is used because the comparison between entities k and j in equation (11), that is, the second term on the right-hand side of equation (11) is still transitive with respect to entities within the rolling window. If the index number of entity *j* is free from significant chain drift, so is that of entity k.

That being said, the intransitivity of RWGEKS and RWCCD still poses an empirical challenge: the index values are not uniquely determined but vary with choice of the link entity *j*. In time series context, alternative link periods for calculating the price level for the new period τ has been proposed, including period $\tau - 1$ (Ivancic, Diewert, and Fox 2011; de Haan and van der Grient 2011), a simple geometric mean of all available link periods from $\tau - \kappa$ to $\tau - 1$ (Diewert and Fox 2017), a period between $\tau - \kappa$ and $\tau - 1$ that is most similar to τ in price and quantity (Diewert and Fox 2017), and a weighted geometric mean of all previous periods as link periods with the weights proportional to the share of expenditures matched between period τ and each link period (Melser 2016).

In panel data, in addition to choice of the link period, the researcher needs to decide which entity in the chosen link period should be the link entity. Following Diewert and Fox (2017), a sensible choice of *j* is the entity that is most similar to entity *k* in purchasing patterns. Because comparison of the new price level P_{RWGEKS}^{0k} with the link entity price P_{RWGEKS}^{0j} is transitive with respect to entities in the rolling window, this approach ensures that at least Walsh's multiperiod identity test is satisfied at entities *j* and *k*.⁶ As such, if

product-level prices and sales at entity *j* are identical or proportional to those at entity *k*, then P_{RWGEKS}^{0k} will be equal to or of the same proportion to P_{RWGEKS}^{0j} (Diewert and Fox 2017).

For a balanced and low- or mediumfrequency panel, entities j and k could be the same location in periods $\tau - 1$ and τ , respectively. For an unbalanced panel where purchase data at entity k's location are not available for period $\tau - 1$, j could be a different location that, for example, shared the highest number of items with entity k. Alternatively, one could apply the relative price similarity measures proposed by Diewert (2009) to search for the link entity jbetween periods $\tau - \kappa$ and $\tau - 1$ that most resembles entity k in prices and sales.

Although RWGEKS and RWCCD indexes are not transitive over time except with the link entity, they can be spatially transitive if the price levels of all entities in a period are calculated using the same link entity. For example, suppose P_{RWGEKS}^{0i} and P_{RWGEKS}^{0k} are price levels for entities *i* and *k* in the same period. By linking with price level of the same entity *i*, the price index ratio between *i* and k is the same whether they are compared directly or indirectly through any entities within the rolling window. Moreover, by increasing the width of the rolling window, RWGEKS and RWCCD indexes can be made closer to being transitive over the entire set of entities. The intuition is that a wider rolling window increases the proportion of entities that overlap in the creation of index values for any two entities. When the overlap is perfect, price comparison between the two entities becomes exactly transitive over elementary entities. In the extreme case where the width of the rolling window and the base period are both equal to the time dimension of the panel, RWGEKS and RWCCD indexes become the standard multilateral GEKS and CCD indexes that are exactly transitive.

It is important to recognize that transitivity is not the only criterion against which a panel index could be benchmarked. Another useful index property is characteristicity, which refers to "the degree to which weights are specific to the comparison at hand" (Caves, Christensen, and Diewert 1982). For example, a bilateral comparison between entities iand k using Fisher ideal index is perfectly characteristic because it uses expenditure shares in i and k exclusively as weights;

⁶ Adapting notations of equation (11), Walsh's multiperiod identity test requires: $P_{RWGEKS}^{0j} \times P_{RWGEKS}^{ik} \times P_{RWGEKS}^{k0} = 1$ where $P_{GEKS}^{ik} = \prod_{l \in \{l_{t-\kappa,t}^{l} \cap l_{t-\kappa,t}^{l}\}} (P_{F}^{ll})^{1/N_{t-\kappa,t}^{ik}}$ and $P_{RWGEKS}^{k0} = 1/P_{RWGEKS}^{0k}$, the latter of which follows from Fisher ideal and Törnqvist indexes' consistency with the time reversal test.

comparing price levels between i and k using GEKS is less characteristic because data from potentially less relevant entities are also used to create the index. Unfortunately, as Drechsler (1973) concluded: "There is no perfect solution since characteristics and circularity [i.e., transitivity] are always ... in conflict with each other." In the context of rolling-window panel indexes, although increasing width of the rolling window improves transitivity, it decreases characteristicity by bringing data from more distant past into the comparison.

Empirical Illustration

In this application, we use county-level price indexes to make cost of living adjustment to food benefits issued by SNAP. SNAP is the largest U.S. domestic food assistance program. In FY 2016, it served over 44 million low-income participants at an expense of \$71 billion, \$67 billion of which was food cost. The maximum SNAP benefit is set equal to or a multiple of the cost of food under the USDA's Thrifty Food Plan-a model food basket that delivers a nutritious diet at minimal expense. The actual benefit amount is an inverse function of net household income, which provides the maximum benefit amount to households with zero net income (Wilde 2001). The Food and Nutrition Service (FNS) of USDA makes (temporal) cost-of-living adjustments to maximum benefit amount, and other program parameters at the beginning of each federal fiscal year. However, the maximum benefit amount is the same across all states and territories in the contiguous United States. There is concern that the geographically uniform maximum SNAP allotment may be inadequate if regional food price varies substantially (Leibtag 2007). In 2013, the Institute of Medicine (IOM) and National Research Council (NRC) Committee on Examination of the Adequacy of Food Resources and SNAP Allotments recommended that "the USDA should examine possible approaches to account for geographic variation in food prices, for example by adjusting the maximum benefit amount to account for price adjustments in high- and low-cost regions of the nation," (Institute of Medicine 2013).

County-Level Price Indexes

The retail scanner data used in this case study are Nielsen Scantrack data from 10,305 participating supermarkets in 1,491 counties collected over 53 quadweeks (i.e., periods of 4week aggregates) starting on April 7, 2002, and ending on April 15, 2006. Universal Product Code (UPC)-level sales data are collected at the point of sale and transmitted to Nielsen. Items can be defined at the UPC or more aggregated levels. We defined an item as a unique combination of brand, size, and multipack (multi). The brand field in Nielsen ScanTrack is very detailed and specific. For example, Coca-Cola Classic Regular, Coca-Cola Cherry Regular, Coca-Cola Vanilla Regular, Coca-Cola Caffeine Free Regular, Coke Zero, and Diet Coke are six different brands in Nielsen data. This level of disaggregation probably preserves the most important differentiation of products as perceived by consumers. From the computational standpoint, this approach saves CPU processing time by reducing the number of elementary products in the price index creation by about 50% compared with directly using UPC level data. In the end, we have 11,811 unique beverage products.

We aggregated store-level data to the level and calculated Paasche, county Laspeyres, Fisher ideal, Törnqvist, RWGEKS-1, RWGEKS-13, RWCCD-1, and RWCCD-13.8 The latter four indexes are variants of the panel RWGEKS and RWCCD indexes. The RWGEKS-1 and RWGEKS-13 are rolling-window GEKS indexes, where the suffix indicates the number of preceding periods included in the rolling window. Therefore, RWGEKS-1 and RWGEKS-13 have a rolling window of 2 and 14 quadweeks, respectively. RWCCD-1 and RWCCD-13 are similarly defined. In this beverage example,

 $^{^{7}}$ The distribution of product counts is as follows: regular carbonated soft drink (1,822), diet carbonated soft drink (754), fruit drink (1,567), fruit juice (2,393), whole milk (915), low fat milk (839), bottled water (2,337), sports and energy drink (618), and bottled tea (566), where count is in parentheses.

⁸ Aggregation to the county level creates county-level unit values by brand, multi, and size. To the extent that retailers differentiate in the amenities (e.g., return policy, number of checkout counters, availability of self-checkout) offered to customers, aggregation over retailers may result in a bias. However, defining elementary products at the retailer-brand-multi-size level creates a severe matching problem for the spatial aspect of panel price comparison: because not all retailers operate in all counties, a large number of bilateral indexes used as inputs into the panel in dexes will have no match at all.



Figure 1. Beverage price index numbers by county (year 1 national mean = 1). *Note*: Scanner data for San Francisco County were not available until the 9^{th} quadweek of the 53-quadweek sample

an entity is a unique pair of county and quadweek.

To find a link entity *j* for entity *k*, we gave priority to the entity in the same location as k but in period $\tau - 1$. If this entity did not exist, we then selected the entity that first was temporally closest to k, then shared the largest set of products with k, and finally had the highest sales of the common set of products. We did not employ a formal relative price similarity index such as those of Diewert (2009) to search for the link entities because it would have increased the processing time enormously for such a large panel. Because sales data are not available in all counties in all periods, we have an unbalanced panel with 76,357 entities for each index. These entities account for 82% of SNAP allotment and 81% of SNAP population in the contiguous United States during the sample period.

Figure 1 plots the index numbers for four select counties (King County, WA; Los Angeles County, CA; Philadelphia County, PA; and San Francisco County, CA) using the eight index formulae. The two superlative bilateral indexes and the four panel indexes, which track each other closely over time in each county, are bounded from above by the Laspeyres index and from below by the Paasch index, as expected. Figures 2a and b juxtapose the four counties by index formula to highlight the spatial price difference. Over most of the 53-quadweek period, Philadelphia County had the lowest beverage cost while San Francisco County had the highest price. The price levels of Philadelphia and Los Angeles Counties appear to converge in the second half of the sample, while the price level of King County seems to diverge from that of Los Angeles County over time. By the end of the sample, King and San Francisco Counties, and Philadelphia and Los Angeles Counties have more similar prices, respectively.

In recent years since the end of our sample period, an increasing number of local jurisdictions have called for leveraging sugarsweetened beverage taxes to fight the obesity epidemic. As of September 2017, three (King, Philadelphia, San Francisco) of the four illustrated counties have either implemented or approved an excise tax of 1 cent per ounce or higher sugar-sweetened beverages. on Because the effectiveness of these taxes in achieving its public health goals hinges on its ability to raise prices of the taxed products, the scanner data panel indexes described here can be used to monitor price trends in the taxed and neighboring untaxed counties.



Figure 2. (a) Beverage price comparison across four select counties based on bilateral indexes (year 1 national mean = 1). (b) Beverage price comparison across four select counties based on panel indexes (year 1 national mean = 1)

To offer more quantitative insights, table 1 reports the average absolute percentage differences between all possible pairs of index formulas for the full sample and for Years 1 and 4. Over the full sample, the largest average differences are observed between Laspeyres and Paasche at 6.912%, which is consistent with the graphical evidence in figure 1. By comparison, the average absolute percentage difference between the bilateral superlative Fisher ideal and Törnqvist is the smallest at 0.197%. The absolute percentage differences between the Fisher ideal and RWCCD-1/RWCCD-13, which use bilateral Törnqvist as their elements, are about 4 times as much as the difference between the Fisher ideal and Törnqvist. Similarly, the average differences between Törnqvist and

TANT T TANT	nia i annocati agu	T IIIIA T ASMIIA						
Time period	Index				Index			
		Paasche	Fisher ideal	RWGEKS-1	RWGEKS-13	Törnqvist	RWCCD-1	RWCCD-13
All years	Laspeyres	6.912 (3.706)	3.382	3.167 (1 008)	3.107 (1.884)	3.231	3.017	2.956
	Paasche	(061.c)	3.243	3.486	(1.004) 3.518	(1)	(1.922) 3.629	(1.094) 3.660
	- : ;		(1.648)	(1.936)	(1.884)	(1.728)	(2.000)	(1.961)
	Fisher ideal			0.706 (0.691)	0.658 (0)	0.197	0.819 (0.764)	16/.0
	RWGEKS-1				0.254	0.636	0.229	0.382
	RWGEKS-13				(0.424)	(0.653) 0.578	(0.203)	(0.456)
						(0.519)	(0.450)	(0.200)
	1 ornqvist						0.707	0.009
	RWCCD-1						(100.0)	0.245
Year 1	Lasnevres	6.783	3.316	3.324	3.324	3.267	3.277	(0.430) 3.277
	Decele	(4.246)	(2.017)	(2.234)	(2.234)	(1.965)	(2.252)	(2.252)
	raascne		5.174 (1828)	09.150 (1.867)	0.18U (1867)	5.218 (1 897)	5.227) (1 927)	5.221 (1.927)
	Fisher ideal			0.563	0.563	0.151	0.650	0.650
	PWGFKS_1			(0.467)	0.467)	(0.211)	(0.547)	(0.547)
	T-CVITO M VI				(0.000)	(0.451)	(0.208)	(0.208)
	RWGEKS-13					0.535	0.186	0.186
	Törnavist					(0.451)	(0.208) 0.578	(0.208) 0.578
	PWCCD-1						(0.468)	(0.468)
								(0.000)
Year 4	Laspeyres	7.087	3.468	2.974	2.905	3.210	2.752	2.688
	Paasche	(120.5)	(100)	(1./00) 3.834	(1.650)	(1.082) 3.563	(1.701)	(1.033) 4.072
			(1.604)	(2.108)	(2.010)	(1.688)	(2.148)	(2.071)
	Fisher ideal		~	0.886 (0.819)	0.799 (0.663)	0.264 (0.217)	(0.891)	(0.745)
								Continued

Table 1. Average Absolute Percentage Point Differences between Pairs of Indexes

-	-				- +			
I ime period	Index				Index			
		Paasche	Fisher ideal	RWGEKS-1	RWGEKS-13	Törnqvist	RWCCD-1	RWCCD-13
	RWGEKS-1				0.426	0.764	0.266	0.524
					(0.540)	(0.765)	(0.189)	(0.562)
	RWGEKS-13					0.657	0.461	0.246
						(0.590)	(0.573)	(0.183)
	Törnqvist					~	0.856	0.783
	4						(0.808)	(0.631)
	RWCCD-1						~	0.415
								(0.553)

Zhen et al.

Note: Calculated based on county-level price indexes for nonalcoholic beverages. Numbers in parentheses are standard deviations; N = 76,357

RWGEKS-1/RWGEKS-13, which use bilateral Fisher ideal as their elements, are more than 3 times as large as the difference between the Törnqvist and Fisher ideal. The largest average difference between a bilateral superlative index and a panel index is 0.819% between the Fisher ideal and RWCCD-1. The average absolute percentage differences between the rolling-window panel indexes are smaller, ranging between 0.222% and 0.382%, than their differences with the bilateral superlative indexes.

The larger differences between bilateral superlative indexes and rolling-window panel indexes may be caused by structural differences between bilateral and multilateral price comparisons and by temporal changes in the shopping basket that favors the rollingwindow approach as time passes. To understand the relative importance of the two probable causes, it is useful to conduct pairwise comparisons by year. We set the national mean in Year 1 as the base (i.e., T = 13) for all indexes. Because the panel indexes do not have a rolling window in the base year, we can attribute differences in index values in Year 1 entirely to differences in formula. By comparison, we expect changes in shopping baskets to have the largest impact on differences between fixed-basket and rolling-window indexes in Year 4-the last year of our sample.

In Year 1, the average differences between the Fisher ideal and panel indexes are 0.563% for RWGEKS and 0.650% for RWCCD as shown in table 1. In comparison, the average differences between Fisher ideal and panel indexes in the full sample are higher at 0.706%, 0.658%, 0.819%, and 0.791% for RWGEKS-1, RWGEKS-13, RWCCD-1, and RWCCD-13, respectively. These differences in mean differences between Year 1 and full-sample, although small in magnitude, are statistically significant at the 1% level. The results are similar when comparing differences between Törnqvist and panel indexes in Year 1 and the corresponding differences in the full sample. Overall, this suggests that much of the observed differences between the Fisher ideal/Törnqvist indexes and the rolling-window panel indexes can be attributed to structural differences between bilateral multilateral price versus comparisons.

In Year 4, the absolute percentage differences between the Fisher ideal and rollingwindow panel indexes are 0.886% for RWGEKS-1, 0.799% for RWGEKS-13, 1.025% for RWCCD-1, and 0.969% for RWCCD-13. These mean differences are statistically significantly higher than the corresponding Year 1 mean differences. This finding is consistent with the expectation that the rolling-window index numbers would become more distinct from fixed-basket index numbers as time passes. Comparing mean differences between Törnqvist and rollingwindow panel indexes in Year 4 with those in Year 1 yields the same results.

Cost-of-Living Adjustment to SNAP Beverage Allotment

Because only beverage sales data are available to us for this project, we limit the cost-of-living adjustment to the portion of SNAP allotment estimated to be spent on beverages. There are no official statistics on how much SNAP benefits were used toward purchasing beverages in the 2002–2006 period. Therefore, we impute the amount spent on beverages and call it the "SNAP beverage allotment".9 Using the Consumer Expenditure Survey Diary, we calculated that the included beverage types accounted for 14% of total food-at-home expenditures for SNAP participants during the sample period. Assuming SNAP participants spent the same proportion of SNAP benefits on these beverages as their non-SNAP food dollars, we multiply total SNAP allotment by 14% to produce an estimate of the SNAP beverage allotment. Because the USDA does not report county-level SNAP participation and allotment by month, we imputed them using monthly state-level participation and allotment statistics from FNS and July county-level participation estimates from U.S. Census. Supplementary online appendix B describes the imputation procedure in detail.

We adjust beverage allotment for entity k for purchasing power parity as follows

(12)
$$r_snap_i^k = snap^k \times P_i^{0k} \times g_i$$

where $r_snap_i^k$ is the adjusted SNAP beverage allotment for entity k based on price index i, snap^k is unadjusted SNAP beverage allotment, P_i^{0k} is the *i*th price index, and g_i is a multiplier equal to $\{\sum_{k \in I_{1,52}} snap^k\}/$ $\{\sum_{k \in I_{1,52}} snap^k \times P_i^{0k}\}$.¹⁰ As such, g_i ensures the budget neutrality of the cost-of-living adjustment over the full sample.

Table 2 presents the cost-of-living adjustment results by price index and study year. Within each study year, the positive (negative) dollar amount associated with each price index is the sum of SNAP beverage allotment changes in entities that would experience an increase (decline) in benefits due to the cost-of-living adjustment. For example, in Year 1, entities that would gain (lose) under a Laspeyres-based cost-of-living adjustment would receive a total of \$26 million (\$77.8 million) more (less) in SNAP beverage allotment. Because of the budget neutrality constraint, one can verify that under each price index the sum of benefit reductions is equal to the sum of benefit increases over the 4year study period.

The last row of table 2 reports the cumulative effects of cost-of-living adjustments. Interestingly, the total amount of reallocated allotment implied by Fisher Ideal and Törnqvist is within 0.3% of each other. By contrast, adjustments based on RWGEKS-1 and RWGEKS-13 reallocated 4.8% and 5.7% more allotments than Fisher idealbased adjustment, respectively. Again, the difference between a bilateral superlative index and its multilateral derivative is larger than the difference between the bilateral superlative indexes. Similarly, reallocated allotment is 5.9% and 6.7% higher based on RWCCD-1 and RWCCD-13 than the Törnqvist index, respectively. Overall, the percentage differences in reallocated beverage allotment across indexes mirror those percentage differences in index values reported in table 1.

The reallocated allotments in table 2 account for both temporal and spatial costof-living differences as reflected by the respective price indexes. To its credit, FNS already makes a temporal cost-of-living adjustment to SNAP maximum allotments, deductions, and income eligibility standards at the beginning of each federal fiscal year.

⁹ It is important to note that, under current program rules, SNAP-eligible foods can be any food or food product for home consumption, excluding foods sold hot at point of purchase or for on-premise consumption. We coin the term SNAP beverage allotment solely for convenience of discussing the empirical results. It should not be misconstrued as suggesting that SNAP restricts a portion of the allotment to beverage purchases.

 $^{^{10}}$ For this section, we drop the 53rd quadweek so that we have exactly 4 years of data for the cost-of-living adjustment exercise.

	7		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	(00)				
Paasche	Fisher ideal	RWGEKS-1	RWGEKS-13	Törnqvist	RWCCD-1	RWCCD-13	spent on beverages' (in \$1,000)	coverage ratio
+\$37,279	+\$29,812	+\$30,032	+\$29,427	+\$28,508	+\$28,954	+\$28,474	\$2,299,280	0.76
-\$78,176	-\$76,161	-\$81,182	-\$82,238	-\$77,587	-\$82,859	-\$83,716		
+\$37,260	+\$31,892	+\$35,791	+\$34,579	+\$31,234	+\$35,813	+\$34,555	\$2,732,081	0.83
-\$89,158	-\$87,652	-\$90,513	-\$92,891	-\$88,514	-\$91,705	-\$94,028		
+\$90,514	+\$87,683	+\$87,923	+\$91,462	+\$88,160	+\$90,226	+\$93,657	\$3,166,911	0.83
-\$63,052	-\$58,935	-\$64,264	-\$62,865	-\$58,471	-\$64,893	-\$63,279		
+\$117,315	+\$113,631	+\$121,933	+\$122,457	+\$115,857	+\$124,414	+\$124,649	\$3,260,588	0.83
-\$51,982	-\$40,270	-\$39,718	-\$39,932	-\$39,187	-\$39,951	-\$40,311		
\$282,368	\$263,018	\$275,679	\$277,926	\$263,759	\$279,407	\$281,335	\$11,458,861	0.81
	Paasche +\$37,279 -\$78,176 +\$37,260 +\$37,260 +\$37,260 +\$9,158 +\$90,514 +\$117,315 -\$51,982 \$282,368	Paasche Fisher ideal +\$37,279 +\$29,812 -\$78,176 -\$76,161 +\$37,260 +\$31,892 -\$89,158 -\$87,652 +\$90,514 +\$87,683 -\$63,052 -\$58,935 +\$117,315 +\$113,631 -\$51,982 -\$40,270 \$282,368 \$263,018	Paasche Fisher ideal RWGEKS-1 +\$37,279 +\$29,812 +\$30,032 -\$78,176 -\$76,161 -\$81,182 +\$37,260 +\$31,892 +\$35,791 -\$89,158 -\$87,652 +\$37,923 -\$89,158 -\$87,652 +\$37,923 -\$89,158 -\$87,652 +\$37,923 -\$89,158 -\$87,653 +\$87,923 -\$53,052 -\$58,935 -\$64,264 +\$117,315 +\$113,631 +\$121,933 -\$51,982 -\$40,270 -\$39,718 \$282,368 \$263,018 \$275,679	Paasche Fisher ideal RWGEKS-1 RWGEKS-13 +\$37,279 +\$29,812 +\$30,032 +\$29,427 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$78,176 -\$76,161 -\$81,182 -\$82,238 +\$37,260 +\$31,892 +\$35,791 +\$34,579 -\$89,158 -\$87,652 -\$90,513 -\$92,891 +\$90,514 +\$87,683 +\$87,923 +\$91,462 -\$63,052 -\$58,935 -\$64,264 -\$62,865 +\$117,315 +\$113,631 +\$121,933 +\$122,457 -\$51,982 -\$340,270 -\$39,718 -\$39,932 -\$51,982 -\$40,270 -\$39,718 -\$39,932 \$223,368 \$223,018 \$277,926	Paasche Fisher ideal RWGEKS-1 RWGEKS-13 Törnqvist +\$37,279 +\$29,812 +\$30,032 +\$29,427 +\$28,508 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 +\$37,260 +\$31,892 +\$35,791 +\$34,579 +\$31,234 +\$90,514 +\$87,652 -\$90,513 -\$92,891 -\$88,514 +\$90,514 +\$87,683 +\$87,923 +\$91,462 +\$88,160 -\$63,052 -\$58,935 -\$64,264 -\$62,865 -\$584,71 +\$117,315 +\$113,631 +\$121,933 +\$122,457 +\$115,857 -\$51,982 -\$40,270 -\$39,718 -\$39,932 -\$39,187 -\$51,982 -\$64,0270 -\$39,718 -\$39,932 -\$39,187 \$223,368 \$263,018 \$277,926 \$263,759 \$263,759	Paasche Fisher ideal RWGEKS-1 RWGEKS-13 Törnqvist RWCCD-1 +\$37,279 +\$29,812 +\$30,032 +\$29,427 +\$28,508 +\$28,954 -\$77,579 +\$51,161 -\$81,182 -\$82,238 -\$77,587 -\$82,859 -\$77,560 +\$31,892 +\$35,791 +\$34,579 +\$31,234 +\$32,805 -\$891,58 -\$87,652 -\$90,513 -\$92,891 -\$88,514 -\$91,705 -\$891,58 +\$87,683 +\$87,683 +\$87,692 +\$90,226 +\$58,160 +\$90,226 -\$63,052 -\$58,935 +\$817,923 +\$117,315 +\$113,631 +\$121,933 +\$122,457 +\$117,705 +\$30,226 -\$51,982 -\$58,935 -\$64,264 -\$62,865 -\$58,471 -\$64,404 -\$51,982 -\$51,933 +\$117,315 +\$113,631 +\$121,933 +\$122,457 +\$124,414 -\$51,982 -\$64,264 -\$823,932 -\$539,187 -\$39,951 \$539,951 \$2282,368 \$223,759 \$277,926 \$223,77926 \$279,407	Paasche Fisher ideal RWGEKS-1 RWGEKS-13 Törnqvist RWCCD-1 RWCCD-13 +\$37,279 +\$29,812 +\$30,032 +\$29,427 +\$28,954 +\$28,474 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 -\$83,716 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 -\$83,716 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 -\$83,716 -\$78,176 +\$31,892 +\$33,5791 +\$334,579 +\$331,234 +\$335,813 +\$34,555 -\$890,514 +\$87,683 +\$87,923 +\$91,462 +\$88,160 +\$90,226 +\$93,657 -\$63,052 -\$58,935 -\$64,264 -\$62,865 -\$58,471 -\$64,893 -\$63,279 -\$51,982 -\$58,935 -\$64,264 -\$62,865 -\$589,471 -\$64,893 -\$63,279 -\$51,982 -\$58,935 -\$64,264 -\$62,865 -\$589,471 -\$64,893 -\$63,279 -\$51,982 -\$58,931 -\$512,457 \$117,315 \$	Paasche Fisher ideal RWGEKS-1 RWGEKS-13 Törnqvist RWCCD-1 Spent on beverages ^V (in \$1,000) +\$37,279 +\$29,812 +\$30,032 +\$29,427 +\$28,508 +\$28,954 +\$28,474 \$2,299,280 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 -\$82,859 +\$33,716 -\$78,176 -\$76,161 -\$81,182 -\$82,238 -\$77,587 -\$82,359 +\$33,716 -\$78,176 -\$76,611 -\$81,182 -\$82,238 -\$77,587 -\$82,351 +\$34,555 \$2,732,081 -\$890,514 +\$87,683 +\$837,923 +\$91,462 +\$88,160 +\$90,226 +\$93,657 \$2,732,081 -\$80,514 -\$87,652 -\$90,513 -\$891,462 +\$88,160 +\$90,226 +\$93,657 \$3,166,911 -\$66,052 -\$58,935 -\$64,264 -\$62,865 -\$88,160 +\$90,226 +\$93,657 \$3,166,911 -\$66,051 +\$8117,315 +\$113,631 +\$121,933 +\$122,457 +\$115,857 +\$124,414 +\$124,649 \$3,260,588

Zhen et al.

However, the magnitude of adjustments is the same for the 48 contiguous states and Washington, D.C. It is therefore useful to isolate the impact of adjusting for spatial costof-living difference only on SNAP allotment from the overall impact.

Table 3 reports amount of reallocated beverage allotment by study year and price index when the budget neutrality constraint is imposed in every quadweek. In doing this, we ensure that the reallocation is based on spatial cost-of-living difference only. The Paasche-based cost-of-living adjustment generates the highest reallocated allotment at \$261.4 million over the four-year period. The Laspeyres-based reallocation is the lowest at \$239.1 million. Again, the difference between Fisher ideal-based and Törnqvist-based allotment changes is smaller than their differences with the panel indexes that use these bilateral superlative indexes as elements. Finally, comparing total reallocated allotments between tables 2 and 3 by price index indicates that the cumulative reallocated allotment based on spatial price differences is about 90% of that based on both temporal and spatial price differences. This suggests that adjusting for spatial cost-of-living differences vastly outweighs the importance of adjusting for temporal changes in cost of living.

A Comparison with the Quarterly Food-at-Home Price Database

In this section we compare the retail scanner data-based price indexes with price indexes created using data from the ERS Quarterly Food-at-Home Price Database (OFAHPD).¹¹ Unlike our retail scanner data, the QFAHPD is based on purchases reported by a panel of Nielsen Homescan households, which had grown to just under 40,000 households by 2006. The QFAHPD has market-level unit value and expenditure data for 52 food groups in 26 metropolitan markets and 9 nonmetropolitan areas. The advantage of household scanner data is that it is possible to create indexes that are specific

¹¹ It is not meaningful to compare our retail beverage price indexes with the C2ER beverage prices because (*a*) C2ER only collects price data on three beverage items, namely, 2 liter Coca Cola, 64 oz. Tropicana or Florida Natural fresh orange juice, and half-gallon whole milk, and (*b*), without purchase quantities, it is not possible to calculate bilateral superlative and panel indexes based on C2ER.

Study				Allotment re	allocated (in \$1	,000)		
year	Laspeyres	Paasche	Fisher ideal	RWGEKS-1	RWGEKS-13	Törnqvist	RWCCD-1	RWCCD-13
1	\$47,160	\$56,028	\$49,942	\$52,393	\$52,393	\$49,702	\$52,564	\$52,564
2	\$52,845	\$56,629	\$52,810	\$56,698	\$56,766	\$52,855	\$57,423	\$57,503
3	\$73,828	\$74,562	\$71,188	\$74,217	\$74,723	\$71,051	\$75,316	\$75,740
4	\$65,231	\$74,155	\$66,592	\$69,521	\$69,716	\$66,487	\$70,231	\$70,856
Overall	\$239,064	\$261,374	\$240,533	\$252,829	\$253,598	\$240,094	\$255,534	\$256,662

Table 3. Cost-of-Living Adjustment of SNAP Beverage Allotment, Accounting for Spatial Price Difference Only

Note: There are 13 quadweeks per study year. We dropped the 53rd quadweek for the SNAP cost-of-living adjustment analysis so that Year 4 has the same number of quadweeks as other study years. A budget neutrality constraint is imposed in every quadweek so that the sum of county allotment increases is equal to the sum of county allotment decreases in each quadweek.

 Table 4. Summary Statistics: QFAHPD- and Retail Scanner-based Indexes

				P	rice Index			
	Laspeyres	Paasche	Fisher ideal	RWGEKS-1	RWGEKS-4	Törnqvist	RWCCD-1	RWCCD-4
Standard deviation: OFAHPD	0.093	0.089	0.090	0.089	0.090	0.091	0.089	0.091
Standard deviation: Retail scanner data	0.052	0.043	0.047	0.048	0.048	0.047	0.048	0.049
Pearson correlation coefficient ^a	0.608	0.467	0.552	0.546	0.548	0.554	0.547	0.549

Note: This comparison is based on quarterly data. Therefore, the width of rolling window for RWGEKS-4 and RWCCD-4 is 15 months. ^aIndicates that this coefficient measures the degree of correlation between the QFAHPD- and retail-based indexes under each index formula.

to a subpopulation (e.g., lower income). The disadvantage is that prices would be missing if none of the sample households purchased the products. As such, it is not possible to construct county or small-area price indexes based on household scanner data due to insufficient observations. For this comparison, we first aggregated the retail scanner data to the 35 QFAHPD markets by quarter and then recalculated the eight price indexes where an entity is a unique pair of market and quarter, and the elementary product is still defined at the brand-size-multipack level. The QFAHPD-based beverage indexes use unit values of the following QFAHPD beverage groups as inputs: fruit juice, low fat milk, regular fat milk, nonalcoholic carbonated beverage, non-carbonated caloric beverage, and bottled water. Collectively, these groups correspond well to the beverage types covered by the retail scanner data.

Table 4 presents the summary statistics for the quarterly retail- and QFAHPD-based price indexes. Three features are noteworthy. First, QFAHPD-based index values are much more variable—as indicated by their larger standard deviations-than those based on retail scanner data. This is partly because only food group-level unit values and quantities are available from OFAHPD for index construction while the retail-based indexes distinguish goods at the much finer brandsize-multipack level. Neglecting product heterogeneity within food groups may produce significant bias. For example, Handbury and Weinstein (2015) found that eliminating heterogeneity bias caused 97% of the food cost variance among U.S. cities to disappear. Second, the degree of correlation between the retail- and QFAHPD-based indexes is centered at around 0.55, which is not surprising given the material differences between the retail scanner data and QFAHPD. Third, among the eight indexes, the correlation coefficient is highest between the Laspeyres indexes (0.608) and lowest between the Paasche indexes (0.467). This is because Laspeyres uses base quantities that are fixed within each data source as weights while Paasche uses concurrent quantities that fluctuate across entities as weights. The correlation coefficients for the bilateral superlative and panel indexes lie in between as they use a combination of Paasche and Laspeyres indexes as elements.

To explore the economic significance of these differences between data sources, we repeat our earlier SNAP allotment exercise using these quarterly market-level indexes. The upper panel of table 5 reports the total amount of SNAP beverage allotment reallocated by index and data source when accounting for both temporal and spatial price differences. Over the four-year period, between \$352 million and \$376 million in SNAP beverage allotment would be reallocated if one of the QFAHPD-based price indexes is used to make the cost-of-living adjustment. By contrast, the amount reduces to between \$250 million and \$283 million when one of the retail-based indexes is used. The smaller reallocated funds implied by the retail scanner indexes are a direct result of their lower variances.

We now turn to the lower panel of table 5 where only spatial price difference is accounted for by holding the budget neutrality constraint in every quarter. Under each index formula, the retail scanner-based index again implies lower reallocated SNAP allotment than the QFAHPD-based index. The difference in reallocated allotment between QFAHPD and retail scanner data within each index formula is at least three to four times as large as the difference between any two retail scanner data-based indexes.

Summary and Discussion

As price is a key determinant of economic activity and retail scanner data is ever more accessible for researchers, understanding the differences between alternative scanner database price indexes is of increased importance. We compared eight price indexes including four variants of two new panel indexes created from Nielsen retail beverage scanner data. For beverages as a group, the difference between the bilateral superlative and panel index values is small, where the average differences never exceeded 1% in pairwise comparisons. However, even minor differences in index values may compound into larger aggregate effects over time and space. By way of an example, if the USDA had redistributed SNAP benefits in the contiguous U.S. based on county cost of living, the amount of reallocated SNAP beverage allotment would have ranged between \$325 and \$349 million, depending on the type of superlative or panel index used, during the a 4-year sample period.¹² Roughly 90% of the reallocated allotment in this counterfactual exercise is used for achieving spatial purchasing power parity. Given that nonalcoholic beverages represent only 14% of SNAP participants' total foodat-home expenditures, the overall economic impact will be much larger when the total food basket is considered. More broadly, the impact of price differences on purchasing power of program allotment is not limited to SNAP. For example, Çakir et al. (2018) measured the cost of fresh fruit and vegetables using retail scanner data and found substantial variation in the real value of the Women, Infants, and Children Program's fruit and vegetable voucher over time and across cities.

With multiple price indexing methods available for measuring cost of living, a problem facing statistical agencies interested in incorporating scanner data into official price statistics is that the "truth" is not known. There are several ways to potentially address this issue. In time series context, Diewert and Fox (2017) compared competing scanner data price indexes with a benchmark index constructed out of a consumer preference structure assumed to be the truth. The index closest to the benchmark is selected as the winner.

Alternatively, we propose in supplementary online appendix C a more agnostic approach where we calculated eight category-level price indexes for each of nine beverage categories using the retail scanner data. Instead of assuming preferences within each beverage category

¹² Estimates for the continental United States are obtained from dividing total reallocated allotments in table 2 by 0.81 which is the proportion of SNAP population covered by our retail scanner data.

Data source		Allotment	reallocated over	the 4-year Study Pe	riod among the 35 (QFAHPD Marke	ets (in \$1,000)	
	Laspeyres	Paasche	Fisher ideal	RWGEKS-1	RWGEKS-4	Törnqvist	RWCCD-1	RWCCD-4
			Accourt	nting for Temporal	and Spatial Price D	ifference		
QFAHPD	\$376,015	\$351,804	\$362,939	×354,807	\$361,753	\$363,723	\$355,865	\$362,660
Retail scanner data	\$282,632	\$249,794	\$256,917	\$264,550	\$266,036	\$259,203	\$267,029	\$268,348
			A	ccounting for Spatia	Il Price Difference	Only		
QFAHPD	\$302,637	\$278,854	\$289,426	\$290,568	\$291,091	\$290,064	\$291,099	\$291,586
Retail scanner data	\$236,076	\$218,740	\$221,102	\$226,861	\$229,210	\$222,203	\$229,312	\$231,905

are known to the econometrician, we estimated a flexible demand system with nine beverages and a *numéraire* eight times, using one

Amer. J. Agr. Econ.

of the bilateral or panel indexes as prices each time. The rationale is that price indexes most closely approximate consumer preferences should generate the best goodness of fit. Consistent with a priori expectations, we found that the superlative indexes are preferred to the Laspeyres and Paasche indexes, and that the panel indexes outperform the corresponding bilateral superlative indexes that they use as elements.

Regarding the width of the rolling window $\kappa + 1$, although we did not find it to be numerically important in the beverage application, this is certainly specific to the example chosen. In our ongoing effort to create panel indexes for all categories of food at home, we find a one-month rolling window unable to eliminate chain drift in the panel indexes for sugar and other sweets-a category with significant seasonal variation in product variety and pricing (e.g., Valentine's Day, Easter, Thanksgiving, Christmas, New Year). The RWGEKS-1 and RWCCD-1 index values declined from the base of 1 in year 1 to 0.20 and 0.13 by year 5, respectively. Increasing width of the rolling window to 14 quadweeks (i.e., 1 year plus 1 quadweek) eliminated the downward drift. For this reason, we recommend that the RWGEKS and RWCCD be calculated with a one-year rolling window to prevent any possibility of chain drift.

A 13-month rolling window is preferred by Ivancic, Diewert, and Fox (2011) and de Haan and van der Grient (2011) in their adaptation of GEKS and CCD to time series comparison. For time series indexes, the rolling-year GEKS index is only marginally more demanding on computing resources than, for example, a rolling-month GEKS because instead of comparing current prices with those in last month only, a rolling-year GEKS compares with the last 12 months. But the same cannot be said for RWGEKS and RWCCD indexes for large panels. In the beverage application, there are 18,507 entities in the base year and an average of 1,446 entities per quadweek thereafter. To construct the RWGEKS (RWCCD) for entities in the base year, one needs 171,263,778 bilateral Fisher ideal (Törnqvist) index values. After the base year, one needs 3,135,651 and 28,226,643 bilateral index values per quadweek to create RWGEKS-1 and RWGEKS-13,

on 24 August 2018

respectively.¹³ With the enormous number of bilateral comparisons needed for RWGEKS and RWCCD, the amount of computing time for determining the lists of common products sold in all pairs of entities can be substantial.

Therefore, unless an efficient canned routine is readily available, it would be tempting for practitioners to adopt a narrow rolling window to speed up calculation; but doing so would unintentionally subject the index values to potential chain drift for strongly seasonal goods. To assist future applications of RWGEKS and RWCCD indexes, we provide a SAS routine in the supplementary appendix online for interested users. Users have the options to specify the length of the base period and width of the rolling window. The SAS codes leverage the multi-core processing capability of any modern computer to calculate the numerous bilateral elementary indexes in parallel and, thereby, achieve tremendous savings in computing time in contrast to a routine that only uses one core of the CPU.¹⁴

Concluding Remarks

There are at least three areas for further research. First, as is the case in time series adoption of rolling window multilateral indexes, the panel RWGEKS and RWCCD indexes are not fully transitive-a necessary tradeoff in creating nonrevisable indexes with a degree of characteristicity. We have discussed how RWGEKS and RWCCD can approximate full transitivity by increasing the width of the rolling window, or be spatially transitive if all entities in the same period use the same link entity to splice with earlier index numbers. The performance of alternative window width and splicing methods in panel data should be formally evaluated. Without further analysis, the lack of full transitivity remains an important limitation of the rolling window approach.

Second, RWGEKS and RWCCD use the matched-item approach to control for product quality differences. An alternative approach is to use hedonic regressions to quality-adjust prices. If consumers perceive identical products marketed by higher- and lower-end retailers to have different quality, it might be desirable to treat them as different products in the index calculation. Because not all retailers are present in all locations, this may cause a lack of productlevel matches across space. To avoid this, the researcher could use hedonic regressions to remove retailer heterogeneity in productlevel prices before constructing the index.

Third, by using matched items in elementary bilateral comparisons, the panel **RWGEKS** and **RWCCD** ignore the quality effect of unmatched items on the index number. The lack of matching is due to product entry and exit over time and not all products being available at all locations. de Haan and Krsinich (2014) incorporate unmatched items into the rolling-window time series GEKS by imputing their prices during periods they are not available through a hedonic regression model. A more utility-theoretic but more technical alternative to the imputation approach is to formally estimate an item-level structural model of consumer demand and back out the virtual prices for the unavailable items (Broda and Weinstein 2010; Handbury and Weinstein 2015). Future work could compare how these two approaches fare in panel data.

Supplementary Material

Supplementary material are available at *American Journal of Agricultural Economics* online.

References

- Albouy, D. 2009. The Unequal Geographic Burden of Federal Taxation. *Journal of Political Economy* 117: 635–67.
- Broda, C., and D.E. Weinstein. 2010. Product Creation and Destruction: Evidence and Price Implications. *American Economic Review* 100: 691–723.
- Bureau of Economic Analysis. Regional Price Parities. https://www.bea.gov/

 $^{^{13}}$ The total count is decomposed as follows. In the base year, by equation (6), a panel index needs 18,507 bilateral comparisons of the artificial base entity with entities in the base year; and 171,245,271 bilateral comparisons for all unique pair of entities in base year. In every quadweek τ after the base year, by equation (11), a RWGEKS-1 (RWGEKS-13) requires 1,044,735 bilateral comparisons between all unique pairs of the 1,446 entities in τ ; and 2,090,916 (27,181,908) bilateral comparisons between entities in τ and the 1,446 (18,798) entities in the preceding 1 (13) quadweek.

¹ ¹⁴ We ran the county-level beverage application on a 12-core server. It took the SAS routine 21.7 hours of CPU time to complete RWGEKS-1 and RWCCD-1; it took another 81.3 hours for RWGEKS-13 and RWCCD-13.

iTable/iTable.cfm?reqid=70&step=1 &isuri=1&acrdn=8#reqid=70&step=1 &isuri=1 (accessed May 15, 2018).

- Çakir, M., T.K.M. Beatty, M.A. Boland, T.A. Park, S. Snyder, and Y. Wang. 2018.
 Spatial and Temporal Variation in the Value of the Women, Infants, and Children Program's Fruit and Vegetable Voucher. *American Journal of Agricultural Economics*.
- Caves, D.W., L.R. Christensen, and W.E. Diewert. 1982. Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers. *Economic Journal* 92: 73–86.
- de Haan, J., and H.A. van der Grient. 2011. Eliminating Chain Drift in Price Indexes Based on Scanner Data. *Journal of Econometrics* 161: 36–46.
- de Haan, J., and F. Krsinich. 2014. Scanner Data and the Treatment of Quality Change in Nonrevisable Price Indexes. Journal of Business & Economic Statistics 32: 341–58.
- Deaton, A.S. 1988. Quality, Quantity, and Spatial Variation of Price. *American Economic Review* 78:418–30.
- Deaton, A.S., and O. Dupriez. 2011. Purchasing Power Parity Exchange Rates for the Global Poor. *American Economic Journal: Applied Economics* 3: 137–66.
- Diewert, W.E. 1976. Exact and Superlative Index Numbers. *Journal of Econometrics* 4: 115–45.
 - —. 1996. Axiomatic and Economic Approaches to International Comparisons. NBER Working Paper 5559, National Bureau of Economic Research.
 - —. 2009. Similarity Indexes and Criteria for Spatial Linking. In *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, ed. D.S. Prasada Rao, 183–216. Cheltenham, UK: Edward Elgar.
- Diewert, W.E., and K.J. Fox. 2017. Substitution Bias in Multilateral Methods for CPI Construction using Scanner Data. UNSW Sydney Working Paper.
- Drechsler, L. 1973. Weighting of Index Numbers in Multilateral International Comparisons. *Review of Income and Wealth* 19: 17–34.
- Dubay, L., L. Wheaton, and S. Zedlewski. 2013. Geographic Variation in the Cost of Living: Implications for the Poverty Guidelines and Program Eligibility.

Urban Institute. Prepared for the Department of Health and Human Services, Office of the Assistant Secretary for Planning and Evaluation.

- Eltetö, Ö., and P. Köves. 1964. On a Problem of Index Number Computation Relating to International Comparisons. *Statisztikai Szemle* 42: 507–18.
- Feenstra, R.C., and M.D. Shapiro. 2003. Introduction to "Scanner Data and Price Indexes." In Scanner Data and Price Indexes, eds. R.C. Feenstra and M.D. Shapiro, 1–13. Chicago: University of Chicago and NBER.
- Fisher, I. 1922. *The Making of Index Numbers*. Boston: Houghton Mifflin.
- Forsyth, F.G., and R.F. Fowler. 1981. The Theory and Practice of Chain Price Index Numbers. *Journal of the Royal Statistical Society* 144: 224–46.
- Gini, C. 1931. On the Circular Test of Index Numbers. *Metron* 9: 3–24.
- Gregory, C.A., and A. Coleman-Jensen. 2013. Do High Food Prices Increase Food Insecurity in the United States? *Applied Economic Perspectives and Policy* 35: 679–707.
- Handbury, J., and D.E. Weinstein. 2015. Goods Prices and Availability in Cities. *Review of Economic Studies* 82: 258–96.
- Hill, R.J. 1997. A Taxonomy of Multilateral Methods for Making International Comparisons of Prices and Quantities. *Review of Income and Wealth* 43: 49–69.
- ——. 2004. Constructing Price Indexes across Space and Time: The Case of the European Union. *American Economic Review* 94: 1379–410.
- IOM. 2018. Supplemental Nutrition Assistance Program: Examining the evidence to define benefit adequacy. Report Brief, January. http://www.nationalacademies.org/hmd/~/ media/Files/Report%20Files/2013/SNAP/ SNAP_RB.pdf (accessed May 15, 2018).
- Ivancic, L., W.E. Diewert, and K.J. Fox. 2011. Scanner Data, Time Aggregation and the Construction of Price Indexes. *Journal of Econometrics* 161: 24–35.
- Jolliffe, D. 2006. Poverty, Prices, and Place: How Sensitive Is the Spatial Distribution of Poverty to Cost of Living Adjustments? *Economic Inquiry* 44: 296–310.
- Konüs, A.A. 1924. The Problem of the True Cost of Living Index. English Translation Published in: *Econometrica* 7 (1939): 10–29.

- Lamboray, C., and F. Krsinich. 2015. A Modification of the GEKS Index When Product Turnover is High. Paper presented at the 14th Meeting of the Ottawa Group, May 2015, Tokyo, Japan.
- Leibtag, É. 2007. Can food stamps do more to improve food choices? An economic perspective-Stretching the food stamp dollar: Regional price differences affect affordability of food. Economic Information Bulletin No. (EIB-29-2), Economic Research Service, U.S. Department of Agriculture, September.
- Melser, D. 2016. Scanner Data Price Indexes: Addressing Some Unresolved Issues. Journal of Business & Economic Statistics doi: 10.1080/07350015.2016. 1218339.
- OECD. 2018. Purchasing power parities (PPP) (indicator). doi: 10.1787/1290ee5aen (Accessed May 15, 2018).

- Szulc, B. 1964. Indexes for Multiregional Comparisons. *Przeglad Statystyczny* 3: 239–54.
- Todd, J.E., L. Mancino, E. Leibtag, C. Tripodo. 2010. Methodology Behind the Quarterly Food-at-Home Price Database. Economic Research Service, Technical Bulletin Number 1926.
- Törnqvist, L. 1936. The Bank of Finland's Consumption Price Index. *Bank of Finland Monthly Bulletin* 10: 1–8.
- U.S. Government Accountability Office (GAO). 2014. School Meal Programs: Implications of Adjusting Income Eligibility Thresholds and Reimbursement Rates by Geographic Differences. Report to Congressional Requesters. GAO-14-557.
- World Bank. 2018. The International Comparison Program. http://www.worldbank.org/en/programs/icp (Accessed May 15, 2018).